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by

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ABSTRACT

In the magnetic annular shock tube (MAST), the driving force is an azimuthal magnetic field produced by a radial current between the concentric cylinders defining the annulus. Since this magnetic field varies inversely as the distance from the axis of the device, there is a variation in the driving pressure across the annulus, which leads to non-uniformities in the flow. This report is concerned with the nature of the non-uniform flow in a MAST with an annular spacing larger than the radius of the inner cylinder.

Experiments were conducted in a MAST with inner and outer radii of 1 and 3 inches. The speed of the disturbance produced in the tube was found to depend upon the polarity of the discharge, being 5.3 cm/μ sec with the center electrode positive and 4.7 cm/μ sec with the inverse polarity. The distribution of current in the tube was determined with probe coils; it too, depended on polarity. With the inner electrode positive the current was largely confined to a thin sheet that was strongly canted with respect to the walls, the inner edge leading. A thicker current sheet that was nearly normal to the tube walls was observed in the inverse polarity condition.

A model is proposed to explain the operation with positive center electrode. The shape of the forward part of the current sheet is calculated by balancing the magnetic pressure behind the sheet with the gas pressure in front; the Newtonian approximation is used for the gas pressure. The resulting shape, which is approximately parabolic, agrees roughly with experiment. It is proposed that the dependence of the flow on polarity is associated with electron emission problems, and that in "positive" operation the ions carry a substantial part of the current.

SECTION I

CURRENT DISTRIBUTION IN A MAGNETIC ANNULAR SHOCK TUBE

The magnetic annular shock tube, MAST[1], is among the most promising devices for producing high velocity plasmas, and considerable research has been devoted to developing it both as a space propulsion device and as an injector for fusion reactors. Nevertheless, very little is known about the exact mechanism by which the magnetic field accelerates the gas. In particular, for a MAST of large radius ratio, the magnetic pressure $B^2/8\pi$ may decrease by orders of magnitude across the annulus and, thus, cannot be balanced by the constant dynamic pressure $\rho u^2{}_s$ associated with a plane normal shock propagating ahead of it[2]. It is the purpose of this note to give a preliminary report of some studies of the shock shape and current distribution in a MAST which may assist in achieving an understanding of the above question as well as others involving the dynamic interaction of a gas and a magnetic field.

A schematic diagram of the apparatus used in the experiments is shown in Fig. 1. The electrodes of the MAST were copper cylinders 36 inches long with inner and outer radii of 1 and 3 inches. The insulators at the ends of the test section were lucite. The 378 μf capacitor bank was connected to produce a current pulse rising in 2μ sec to a value which remained constant for a test time of 12μ sec. For the experiments reported here, the capacitor bank was charged to 5KV which produced a peak current of 165 ka in argon at a pressure of 50μ Hg. The back EMF V_{13} was 1200 v and the arc drop V_{23} was approximately 50 v. We should note that no external fields or pre-ionization were used in the present experiments.

The shock speed was measured by five tungsten u. v. detectors[3] spaced 6 inches apart along the length of the tube. For given operating conditions the shock speeds were reproducible and constant to $\pm$ 3% over the entire 24'' interval spanned by the u. v. detectors. However, an unexpected dependence of the speed on the polarity of the center electrode was observed. The average speeds for positive and negative polarity were 5. 3 and 4. 7 cm/μ s, respectively.

The coils used to probe the magnetic field were constructed of a single turn of nyclad wire mounted on a glass sting. A typical coil is shown in Fig. 2. The design was based on the theory that if the diameter of the wire were made small compared to a mean-free-path, the interaction with the gas could be made negligible. Although the 5 mil wire employed has a diameter only one-tenth the kinetic theory mean-free-path behind the shock, it is still not clear that the objective was completely achieved. There is a

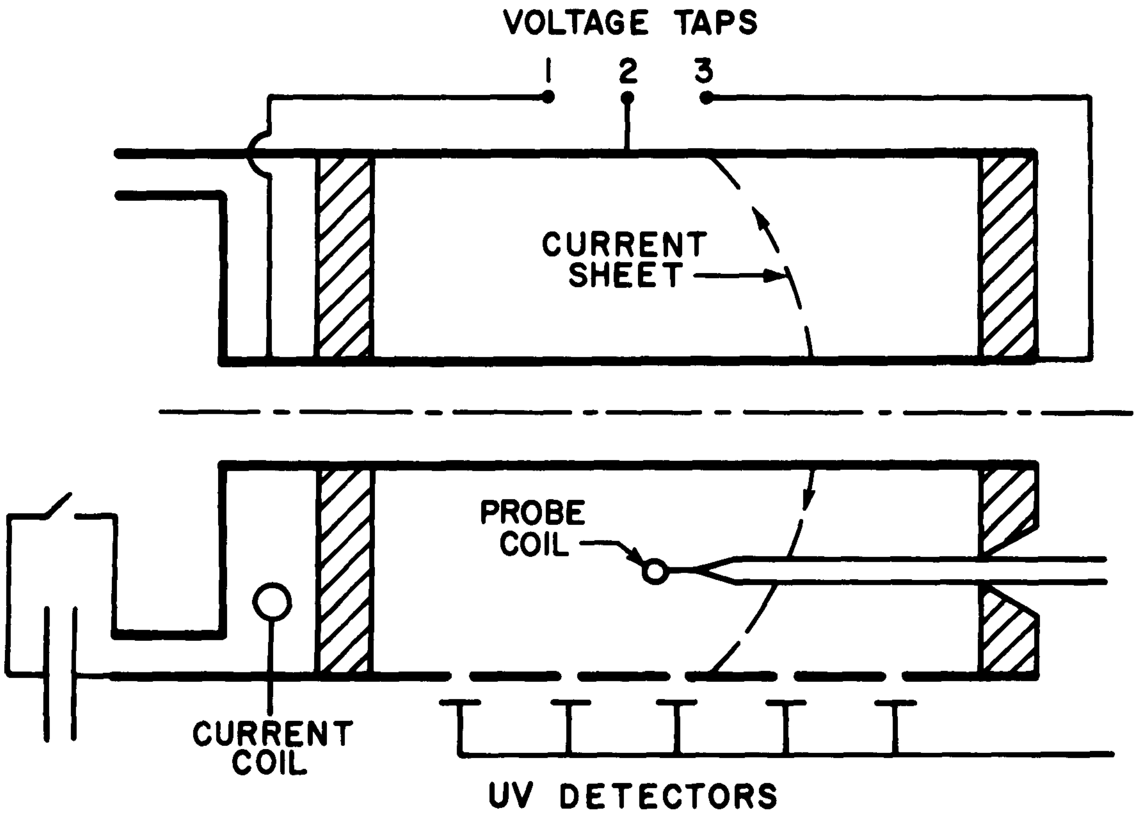

Fig. 1 Schematic diagram of MAST geometry. The total drive current was monitored by the current coil and the magnetic field in the test section was mapped with the probe coil.

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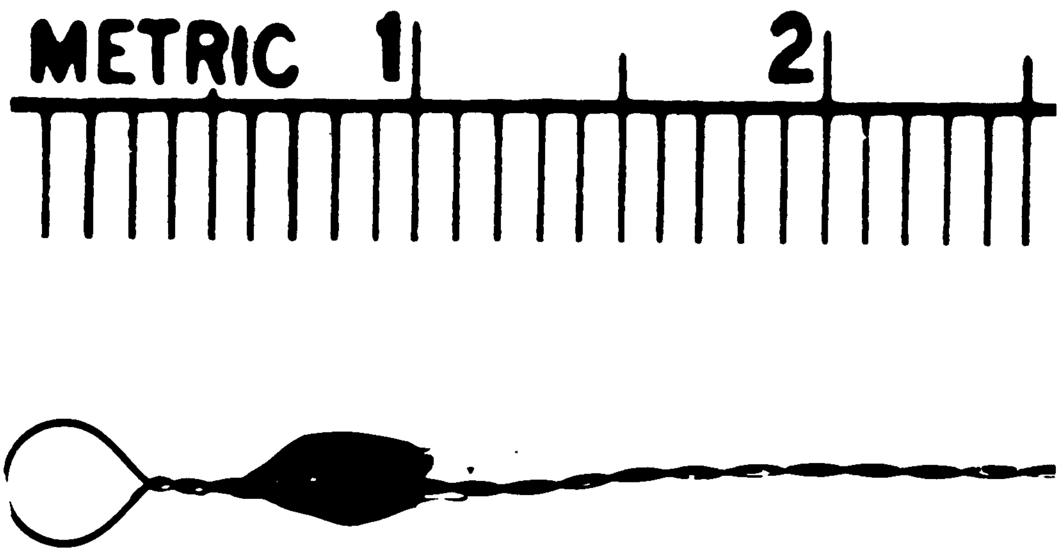

Fig. 2 Photograph of a single turn open coil used to probe the magnetic field.

strong possibility that ablation of the nylon insulation on the wire produces a gas cloud which perturbs the flow. A partly successful attempt to overcome this difficulty by using bare wire to heat-sink the energy has been made by Patrick[3].

In the present experiment, the magnetic field in the MAST was mapped out point by point, by inserting the coil probes as indicated in Fig. 1 to axial positions opposite the u.v. speed detectors and scanning radially in 1 cm steps across the 5 cm gap. Some typical oscillograms of the coil output voltage V_c are shown in Fig. 3. If we assume (as indicated by experiment) that the magnetic field is nearly steady in a coordinate frame moving at the shock speed, then

$$V_c \approx 4\pi \times 10^{-9} \ A u_s j_r \ \text{volts} \qquad (1)$$

where u_s is the shock speed in cm/sec, A is the area of the coil in cm^2, and j_r is the radial current density in amps/cm^2. This equation was used to establish the calibration of the traces in Fig. 3.

It is immediately apparent from an inspection of Fig. 3 that the current pattern in the MAST depends on the polarity of operation. For a positive (outward) radial current, the thickness of the current sheet is $\sim$ 2 cm, while for a negative (inward) current, it is more than twice as thick. Also, as can be seen by comparing the position of the current pulse with the reference signal from the u.v. detectors, the shape of the current sheet changes when the polarity of operation is reversed. For positive operation, the current sheet is severely canted with the inner edge leading the outer edge by 1.5μs in time or 8 cm in space. For negative operation the sheet, although not as well defined, is more or less perpendicular to the walls.

A more quantitative picture of the current distribution is given in Fig. 4. Here we show contours of constant linked current $I = 5 B_\theta r$ obtained by RC integration of the signals in Fig. 3 at two instants in time for both positive and negative operation. All the features of the leading current sheet mentioned above are clearly seen in Fig. 4. An important feature not shown is that there exists a B_θ ahead of the current sheet even though none was applied. This B_θ has a value of almost exactly 1/10 the initial field jump and is associated with an approximately constant current per unit electrode length established in the first fraction of a microsecond after initiation of the discharge during which time the conductivity of the gas is presumably rising to a high enough value to exclude the field.

The development of the current pattern behind the current sheet can also be seen in Fig. 4. For the case of positive operation, the current distribution suggests the growth of a gas "bubble" on the outside wall behind the current sheet. (In this connection, note that the direction of the magnetic pressure is down the gradient of the contours). For the case of negative operation most of the drive current flows at the shock front. A tentative explanation of the observations presented in this note is given in the following letter.

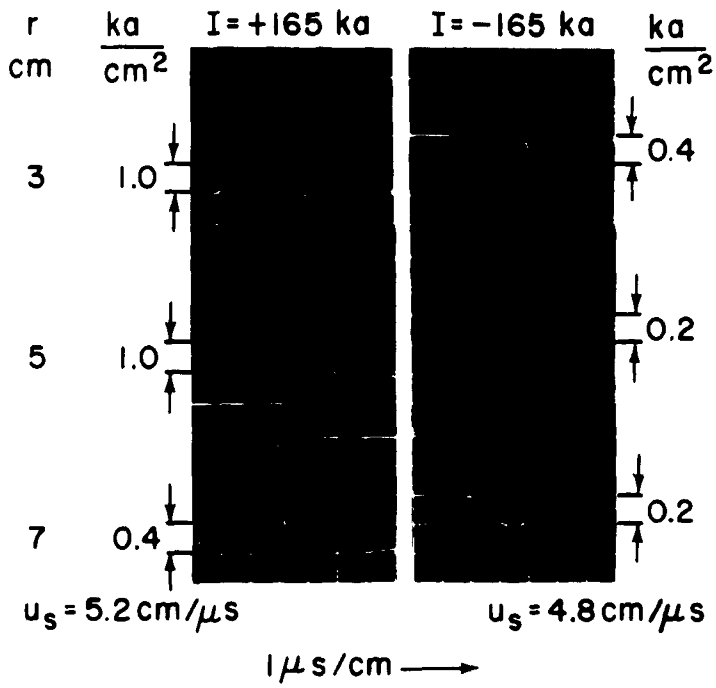

Fig. 3 Oscillograms of the MAST current density at the center of the annulus at an axial station 60 cm from the driver end for various radial positions and positive and negative center electrode. The lower trace in each oscillogram is the current density; the upper trace is a reference u. v. signal obtained from a detector looking across the annulus at the coil position.

In concluding, we should note that our experimental results for a negative center electrode are in general agreement with those reported by Burkhardt and Lovburg[4]. However, the theory they present does not predict a difference in MAST operation associated with polarity. The polarity effects not only the shock speed and current distribution as reported here but also the electrostatic potential and luminosity of the gas. A fuller report will be given in a later paper.

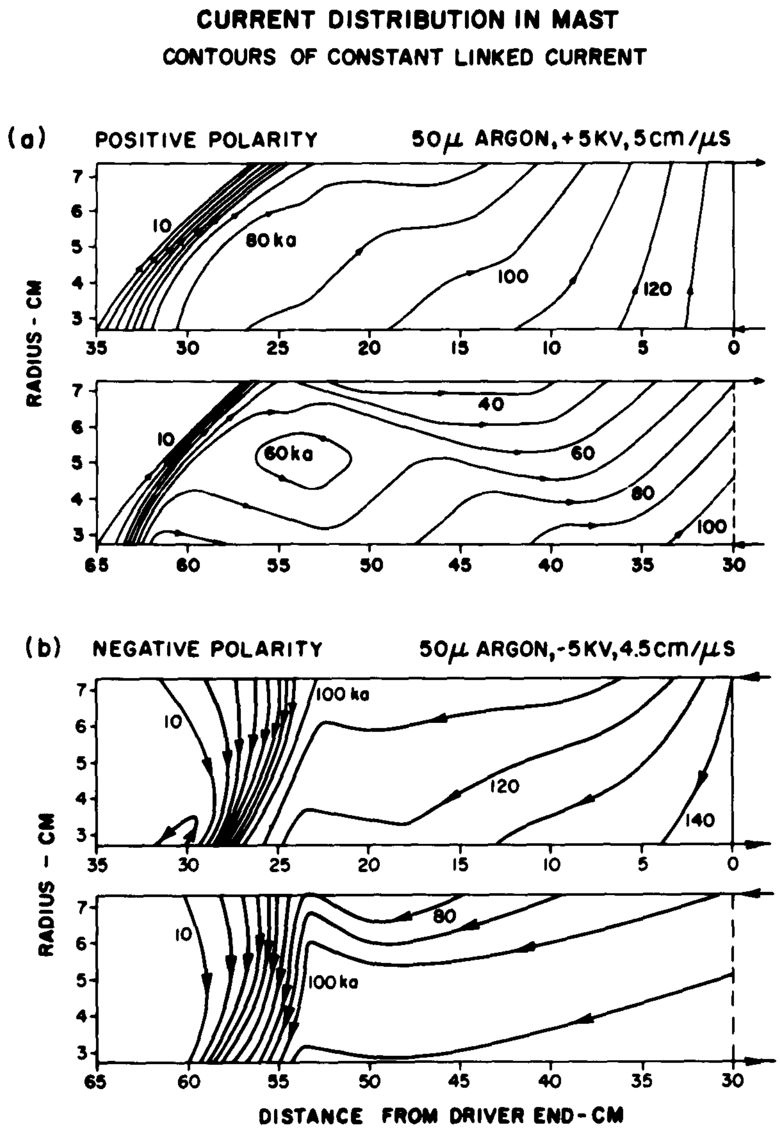

Fig. 4 Contour plots showing current distribution in the MAST for positive and negative center electrode. Note that the axial scale of the figure has been compressed by a factor of 2.

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SECTION II

A FLOW MODEL FOR LARGE RADIUS-RATIO MAST OPERATION

In this note we propose a model that explains some features of the behavior of large radius ratio MASTS as reported by Keck in the preceding note. Topics discussed are the shape of the current sheet when the center electrode is positive, the speed of this current sheet, and the difference between positive and negative operation.

We first consider a model based on the assumptions: (1) the gas is sufficiently conducting so that the magnetic field does not penetrate into the plasma, i.e., the current is confined to a thin sheet forming the boundary between gas and field; (2) the current flows freely from plasma to electrodes. The total pressure must be continuous across a current sheet, hence, the gas pressure on one side balances the magnetic pressure on the other side. Now the magnetic field varies inversely with radius, so that the magnetic pressure at the inside cylinder greatly exceeds that at the outside. This suggests that the current sheet at the inside is pushed ahead relative to the outside. The model sketched in Fig. 5 is proposed. The current sheet may be regarded as a solid body moving through the gas. A shock wave will form slightly ahead of the current sheet; this shock deflects the flow so that it goes around the current sheet. When this flow reaches the outside wall, its radial motion is stopped and therefore its pressure increased, moving the current sheet further away from the wall and allowing the gas to flow through and form a bubble.

In a coordinate system moving with the current sheet, the forward part of this sheet would be expected to be a steady flow, although the bubble will grow in time. The shape of the steady portion may be computed from the pressure balance condition. In hypersonic flow with a thin shocked layer (as assumed above) the gas pressure at the sheet is approximately the normal component of the momentum flux ahead of the shock[5], i.e.,

$$p = \rho_1 u_s^2 \cos^2 \theta \qquad (2)$$

where ρ_1 is the gas density ahead of the shock, u_s is the flow velocity (i.e., the sheet speed in laboratory coordinates) and θ is angle between the flow velocity and the normal to the sheet. The pressure balance condition becomes

$$\rho_1 u_s^2 \cos^2 \theta = (B_0^2/8\pi)(R/r)^2 \qquad (3)$$

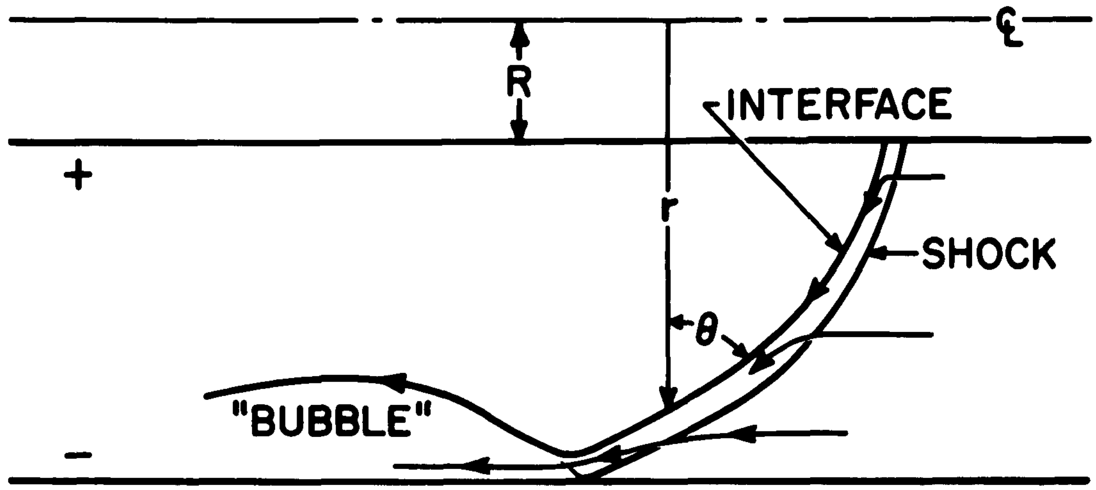

Fig. 5 Flow model for large annulus ratio MAST.

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with B_0 the magnetic field at the inner radius R, and r the local radius. We assume that at R the current sheet forms a right angle with the inside wall. This follows from Eq. (1) if one assumes that at a corner of arbitrary angle the full stagnation pressure $\rho_1 u_s^2$, would be developed. This boundary condition and Eq. (2) determine the sheet propagation speed to be

$$u_s = B_0 (8\pi \rho_1)^{-1/2} \tag{4}$$

If Z(r) is the axial position of the sheet, $\frac{dZ}{dr} = - \tan \theta$, Eq. (2) may be integrated, yielding

$$Z = 1/2\, R \left\{ \frac{r}{R} \left[\left(\frac{r}{R}\right)^2 - 1 \right]^{1/2} - \ln \left(\frac{r}{R} + \left[\left(\frac{r}{R}\right)^2 - 1 \right]^{1/2} \right) \right\} \tag{5}$$

as the equation of the sheet.

The comparison in Fig. 6 of the shape given by Eq.(4) and the experimental results obtained by Keck for positive operation show order of magnitude agreement. The steadiness of this portion of the flow is also evident from the agreement between the interface as observed at a position 60 cm down the tube with the observations at the same conditions but at 30 cm. Figure 4a of the preceding note shows evidence of the bubble formed by the gas that has flowed around the current sheet; the decrease in the magnetic field at the outside wall indicates the presence of gas pressure in that region. The accumulation of gas in this bubble between the 30 cm and 60 cm positions is evident. In Fig. 7, comparison is made of the velocity predicted by Eq. (3) and Keck's experiment[6]. The value of B_0 used in plotting this figure (and also in the code of Fig. 6) was computed from the total current measured at the input leads. Note, however, from Fig. 4a of the preceding note that only about half of this current flows in the current sheet, the rest being distributed in the volume behind this sheet. Thus, for all measured cases, the magnetic pressure directly behind the sheet is less than $\rho_1 u_s^2$. The order of magnitude agreement of the speed, steady slope, and the qualitative indication of bubble growth suggest that this crude model describes some of the basic features of the flow for positive operation.

Nothing in the model discussed above indicates a difference between operation with positive and negative polarity. We suggest this difference is associated with the problem of electron emission at the electrodes. Note that the model as discussed does imply a radially outward mass flow. If the center electrode be positive, the positive ion current associated with this mass flow is adequate (for the range of gas densities considered) to account for all the current in the sheet except near the center electrode. Thus, in this case, electron emission at the cathode is unnecessary. However, for negative operation the ion flow is opposed to the current, hence the latter would have to be carried by electrons. If sufficient electron emission at the cathode is not possible, our model is inappropriate. In this case, a mode of operation may develop involving radially inward mass flow thus providing the

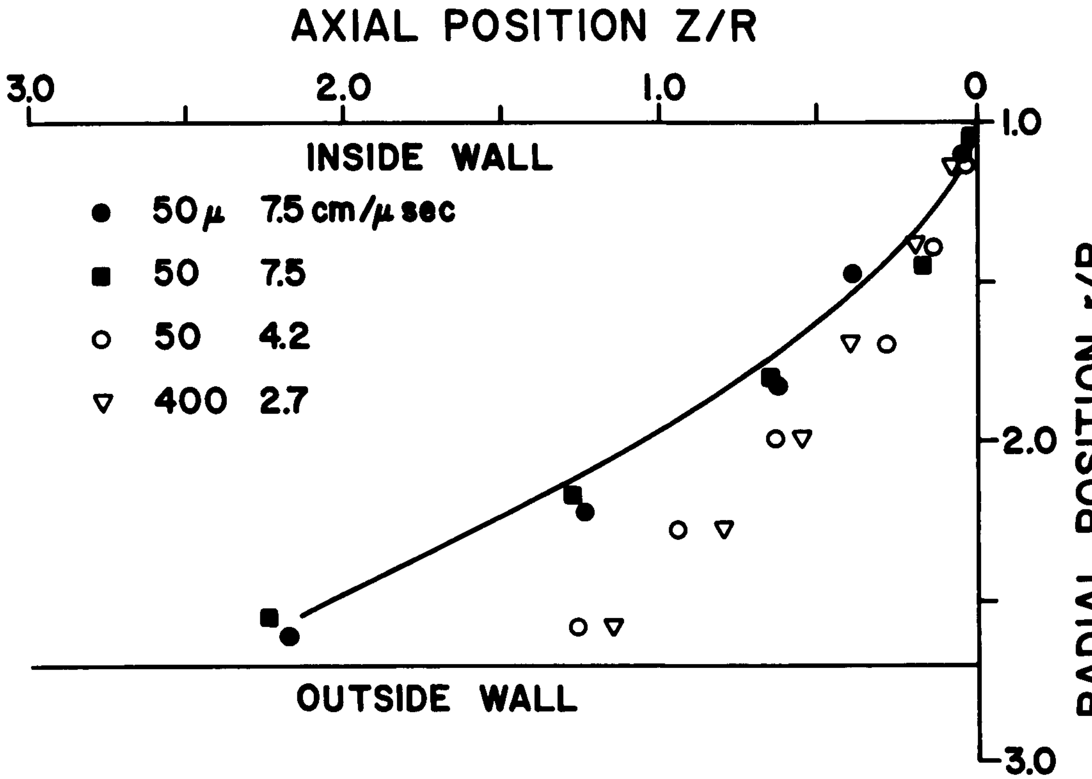

Fig. 6 Comparison of the shape of the current sheet observed by Keck with Eq. (4). The experimental positions were taken to be the points of maximum current density. The code indicates the gas pressure (in microns) in the tube before the discharge and the speed computed from Eq. (3) corresponding to the initial conditions. All observations were made at a station 60 cm down the tube except those represented by the squares, which were made at 30 cm.

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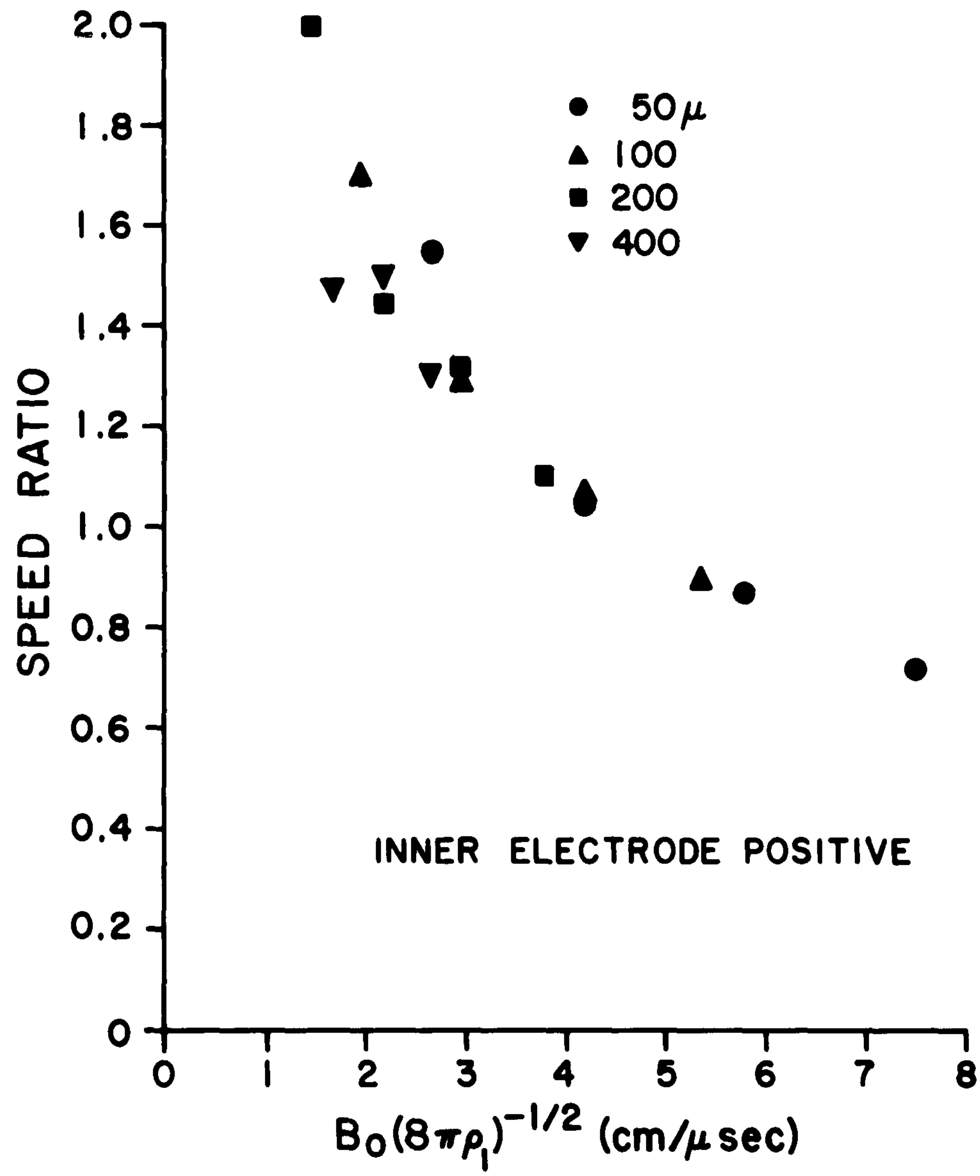

Fig. 7 Ratio of the speed of the current sheet observed by Keck to the speed $u_s = B_0 (8\pi \rho_1)^{-1/2}$ plotted against the latter speed. The points are coded according to the gas pressure (in microns) in the tube before the discharge.

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appropriate ion current for negative operation. The leading current at the outside wall shown in Fig. 4b of the preceding section gives some suggestion of this type of behavior. We would, therefore, suggest that the difference between the positive and negative operation is caused by the fact that there is insufficient electron emission from the electrodes and that this tends to force the flow towards a configuration which allows the current to be carried by the ions.

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Unclassified report

In the magnetic annular shock tube (MAST), the driving force is an azimuthal magnetic field produced by a radial current between the concentric cylinders defining the annulus. Since this magnetic field varies inversely as the distance from the axis of the device, there is a variation in the driving pressure across the annulus, which leads to non-uniformities in the flow. This report is concerned with the nature of the non-uniform flow in a MAST with an annular spacing larger than the radius of the inner cylinder. Experiments were conducted in a MAST with inner and outer radii of 1 and 3 inches. The speed of the disturbance produced in the tube was found to depend upon the polarity of the discharge, being 5.3 cm/μ sec with the

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2. Shock Tubes - Flow.
3. Flow Non-Uniform.
4. Gases - Magnetic Field Effects.

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IV. Petschek, H.
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